Lecture 3:
A Primer on Signals

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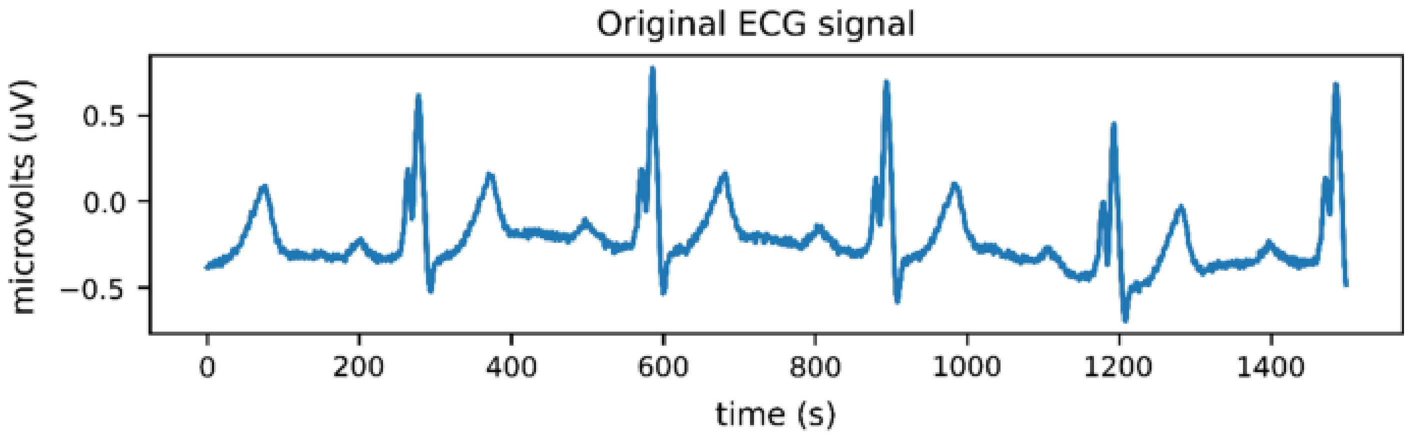
Contents

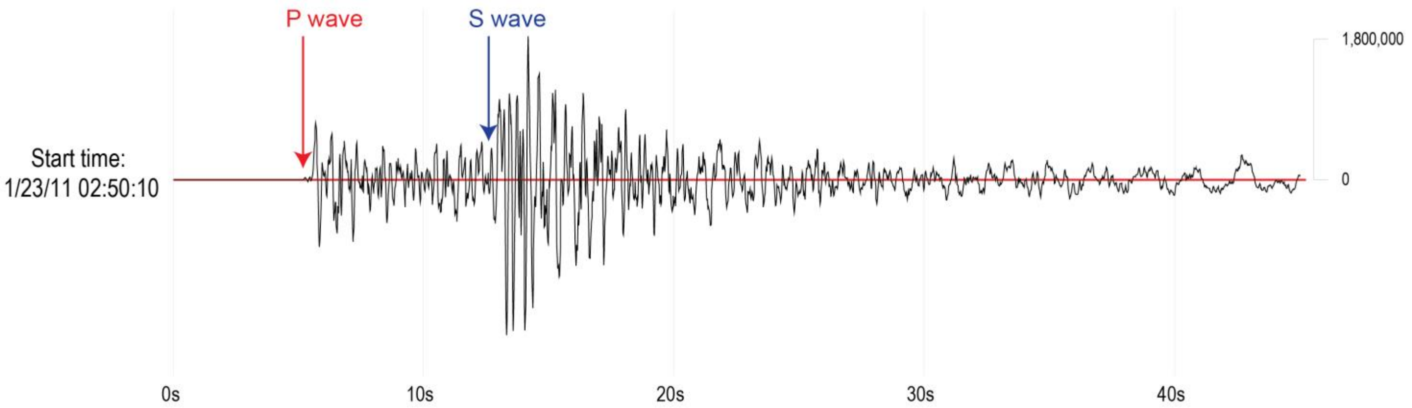
- What are Signals/Data?
- Analog/Digital Signals
- Common Signals: Sinusoids
- Complex Signals
- Sampling
- Time-Frequency Domains

What Are Signals?

- Certain physical variables
  - Voltage, current…
  - Light, temperature, humidity…
  - ECG, blood pressure…
  - Position, velocity, mass, …
  - Price of stocks, interest/exchange rate, …
  - Sound, water ripples, WiFi, infrared, …
  - …
- Time-series data: signals as a function of time (or other variables)
  - The physical variable at a set of times
  - Data!
- Signal Processing / Data Analytics
  - Extract information/contexts from data/signals

Examples


Original ECG signal
microvolts (uV)
0.5
0.0
−0.5
0
200
400
600
800
1000
1200
1400
time (s)

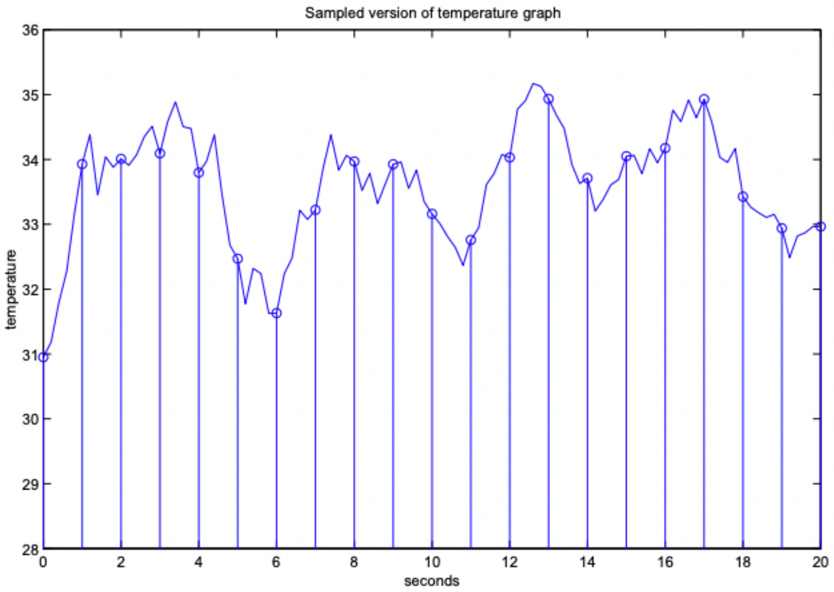

P wave
S wave
Start time:
1/23/11 02:50:10
1,800,000
0
0s
10s
20s
30s
40s

Analog/Digital Signals

- Analog signals: Continuous domain and range
  - Continuous-time signals, x(t)
- Digital signals: Discrete (and often finite) domain and range
  - Discrete-time signals, x[n]

- Digitalize an analog signal (Why?)
  - **Sampling**: Digitalize the (time) domain
  - **Quantization**: Digitalize the range

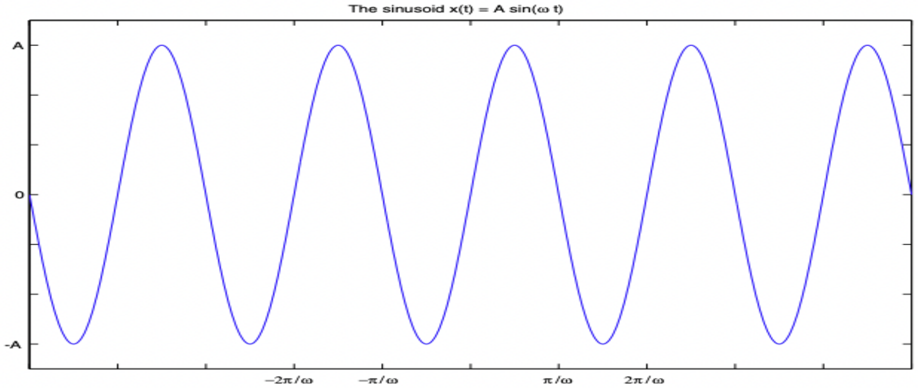
  - Analog-to-Digital Conversion/DAC

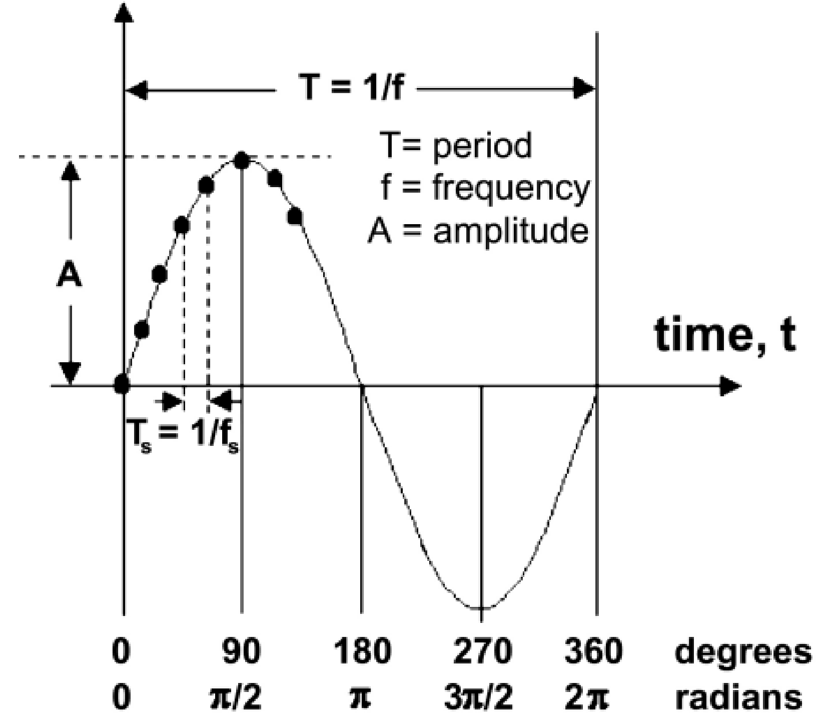


Common Signals: Sinusoids

- $x(t) = A\sin(\omega t) = A\sin(2\pi f t)$
  - A: amplitude
  - ω: radian frequency in radians/s
  - f: frequency in Hertz (Hz) / cycles per sec

- $x(t) = A\cos(2\pi f t - \pi/2)$
  - phase: the position on a waveform cycle
  - angle-like quantity representing the fraction of the cycle covered up to t





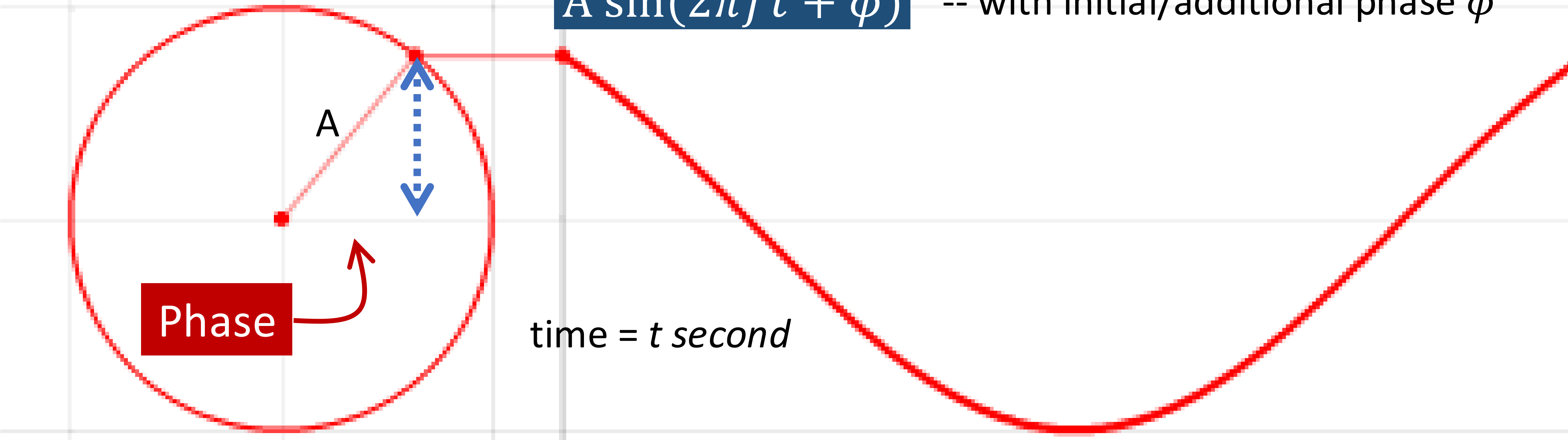
Frequency, Amplitude, and Phase

Frequency, Amplitude, and Phase

2π *angles per cycle*

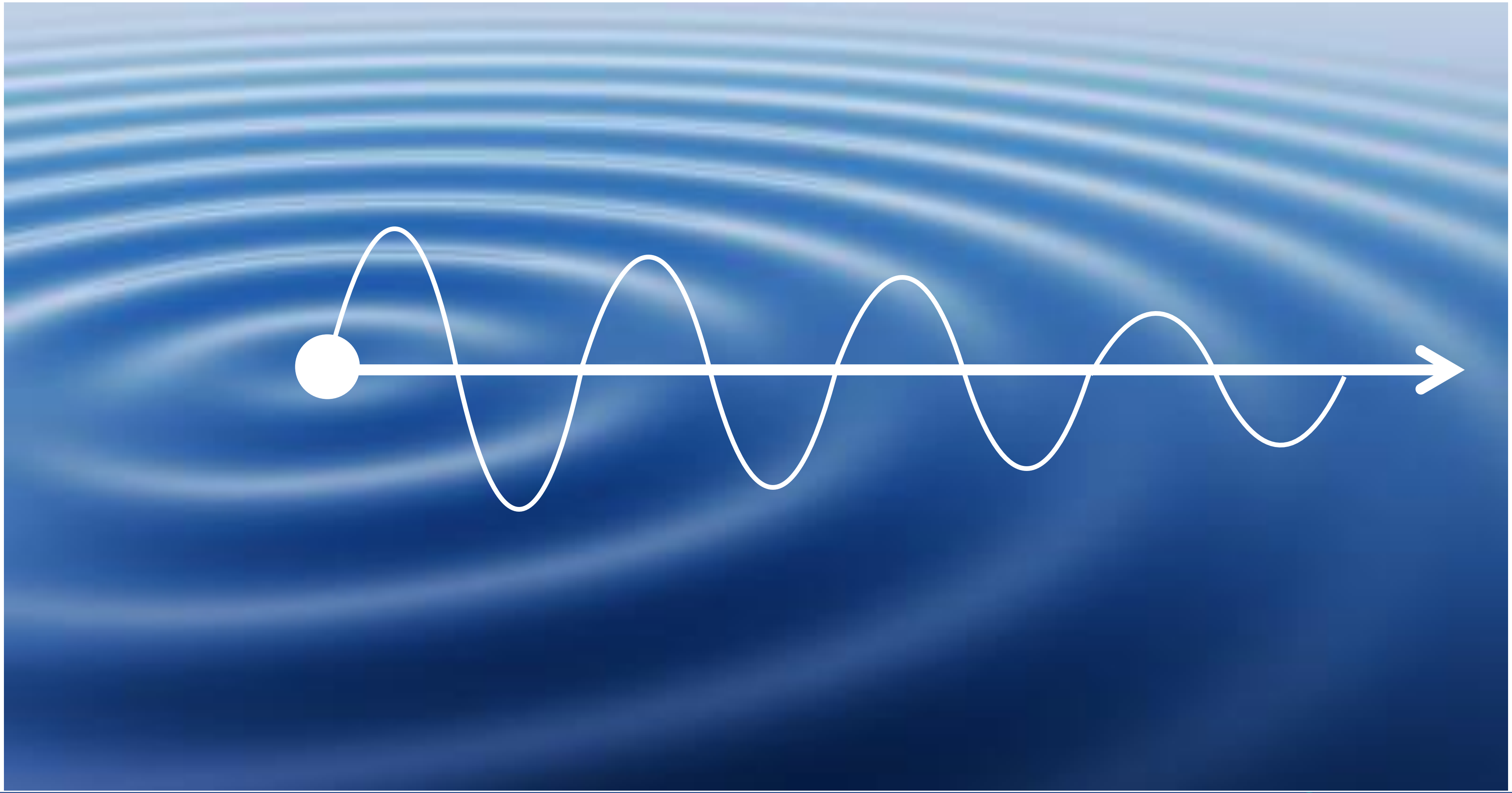
f *cycles per second*

$A\sin(2\pi f t + \phi)$ -- with initial/additional phase ϕ

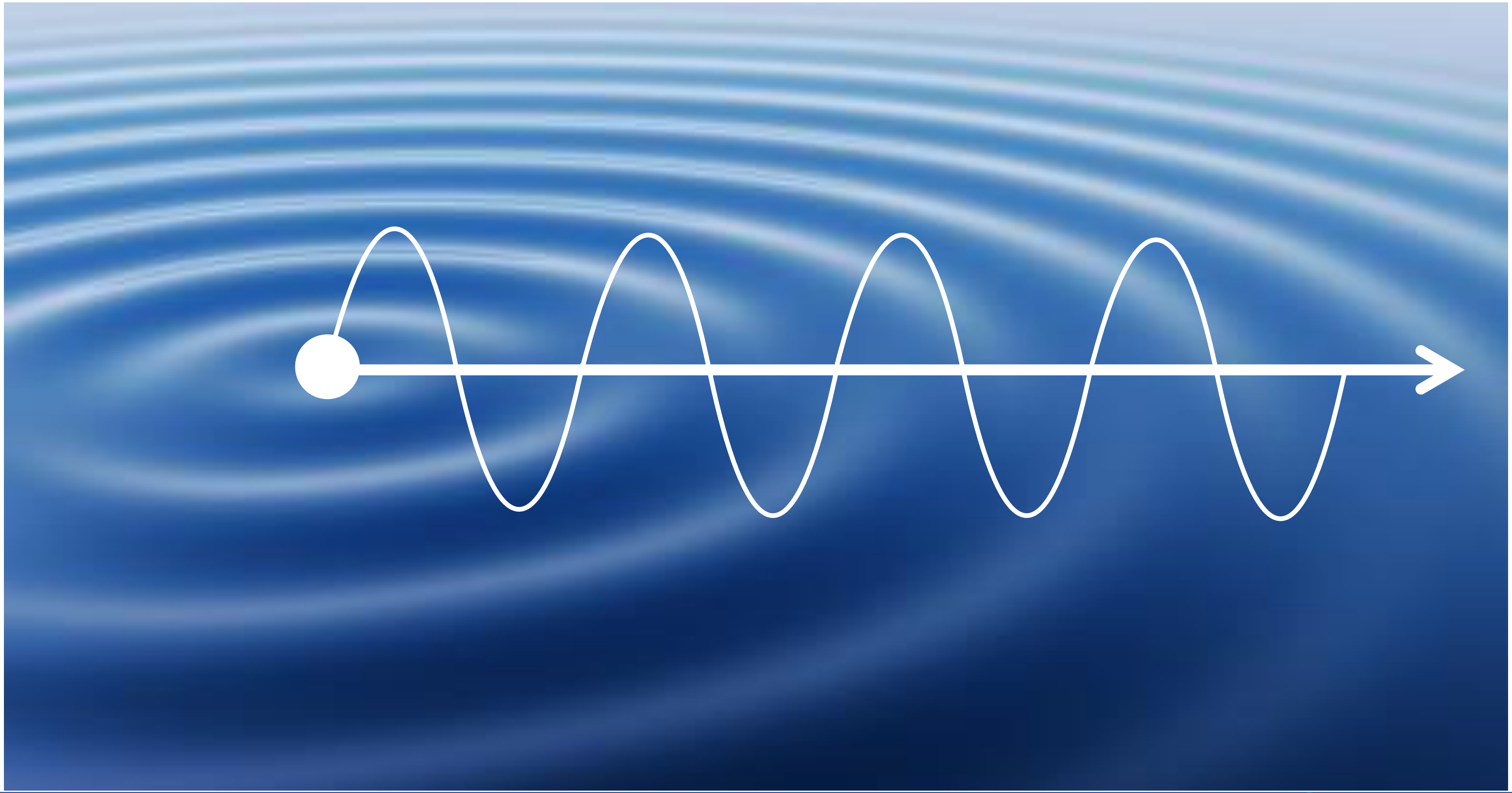


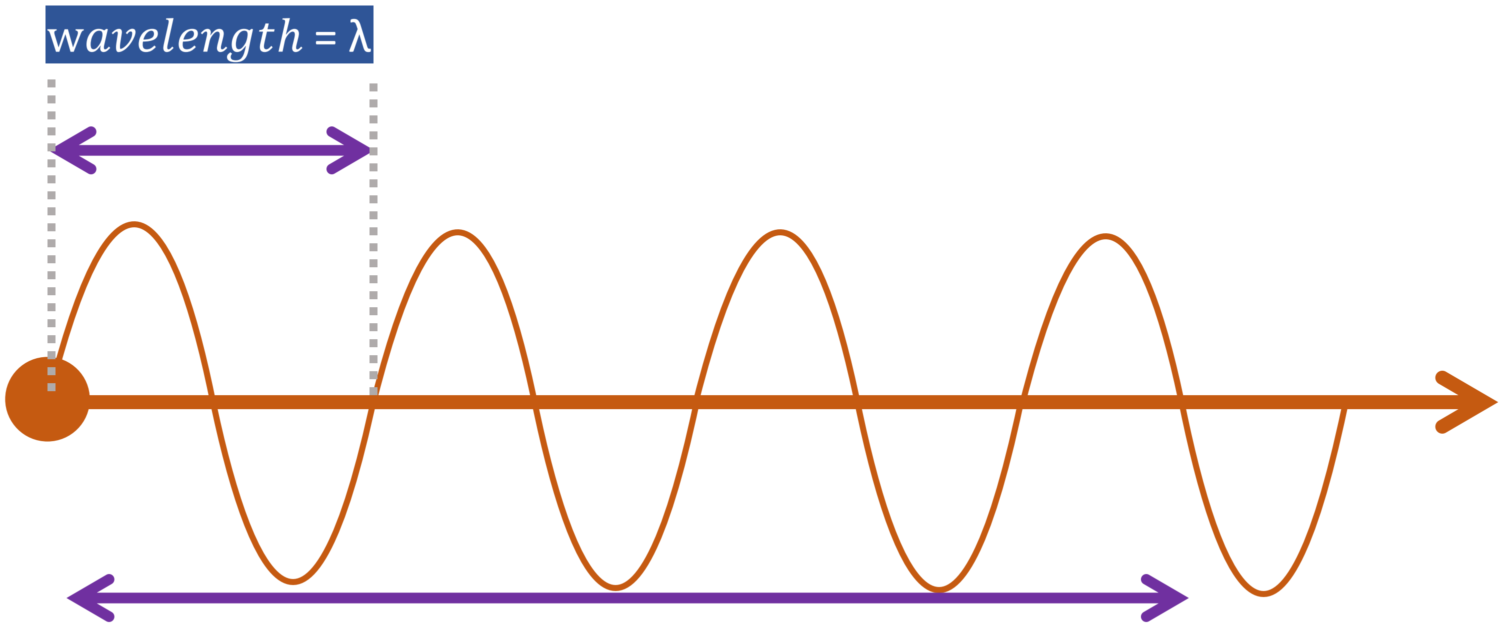
$$x(t) = A\sin(\theta(t)) \;\; = A\sin(2\pi f t) = A\sin(\frac{2\pi}{T} t)$$

Waves in time and space



Waves in time and space





f number of cycles per second

C meters of distance per sec
(speed of the wave)

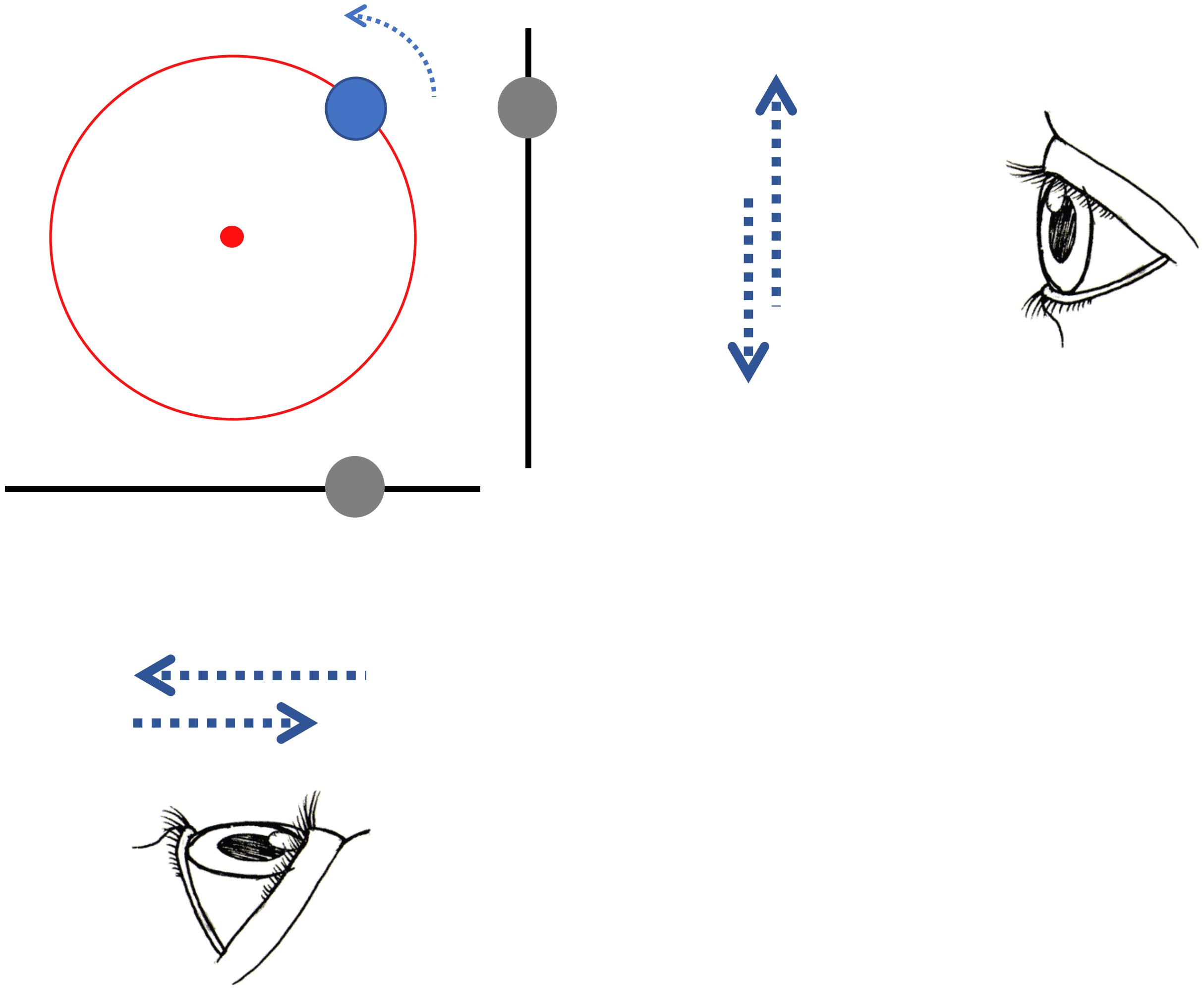
distance per cycle $= \lambda = C / f$

wavenumber $= 1/\lambda$
(cycles per unit distance)

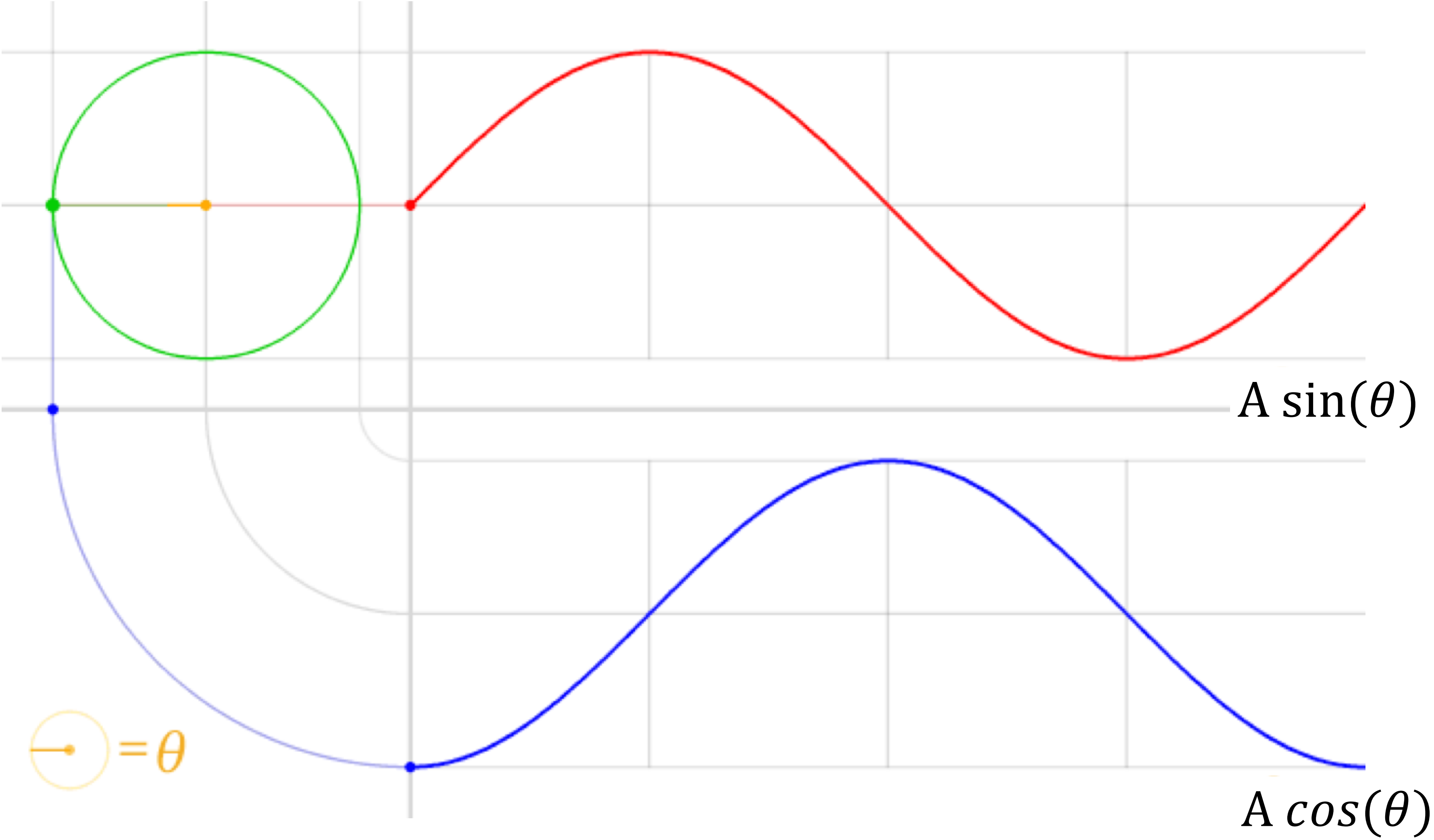
$$x(t) = A\sin(2\pi f t) = A\sin(\frac{2\pi}{T}t)$$

$$x(t, d) = A\sin(2\pi f t - \theta(d))$$

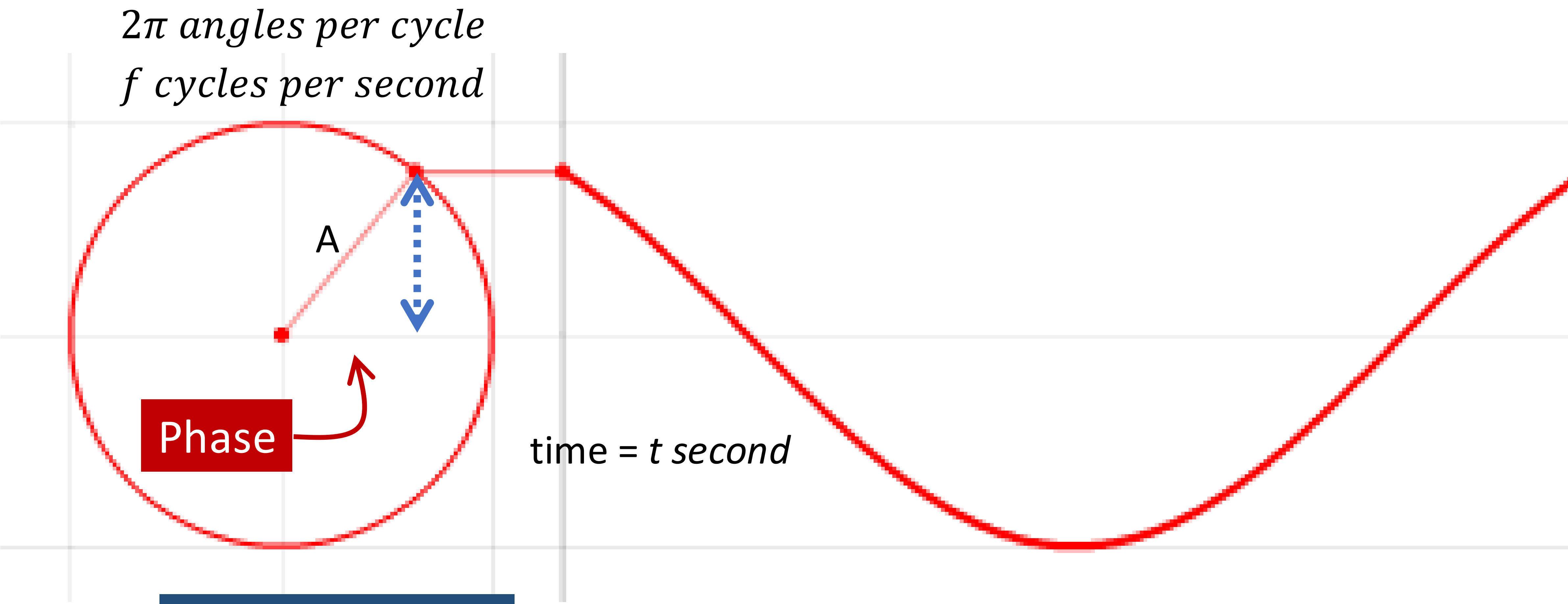
Complex Signals



Complex Signals



Complex Signals

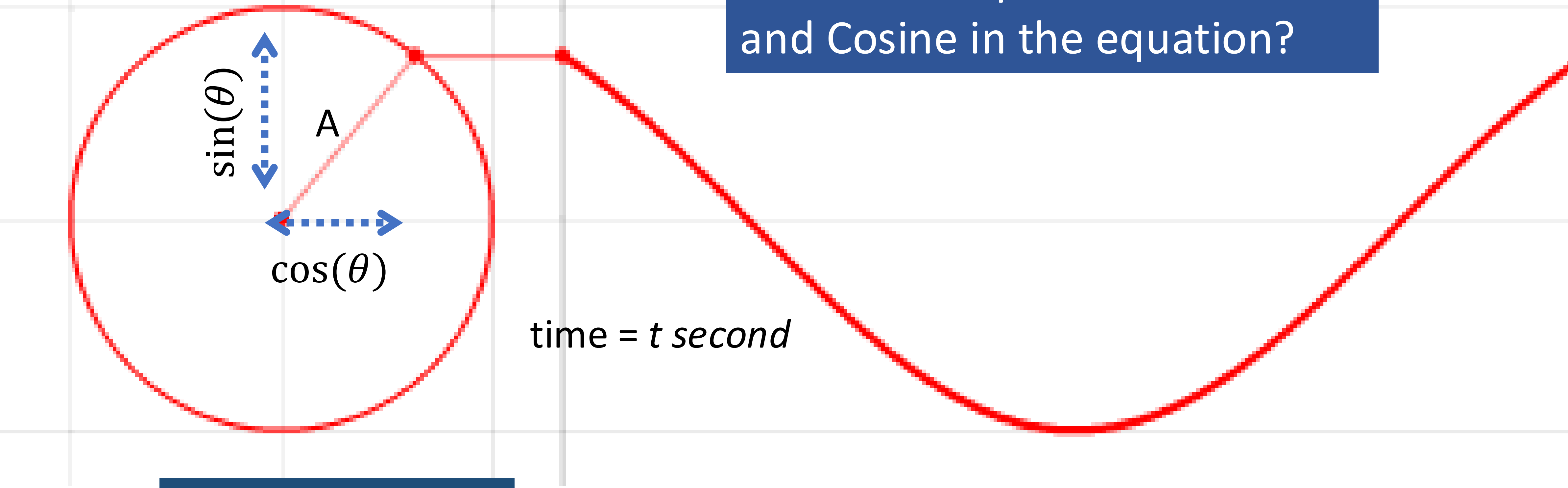


$A\sin(2\pi f t + \phi)$ -- with initial/additional phase ϕ

Complex Signals

2π *angles per cycle*

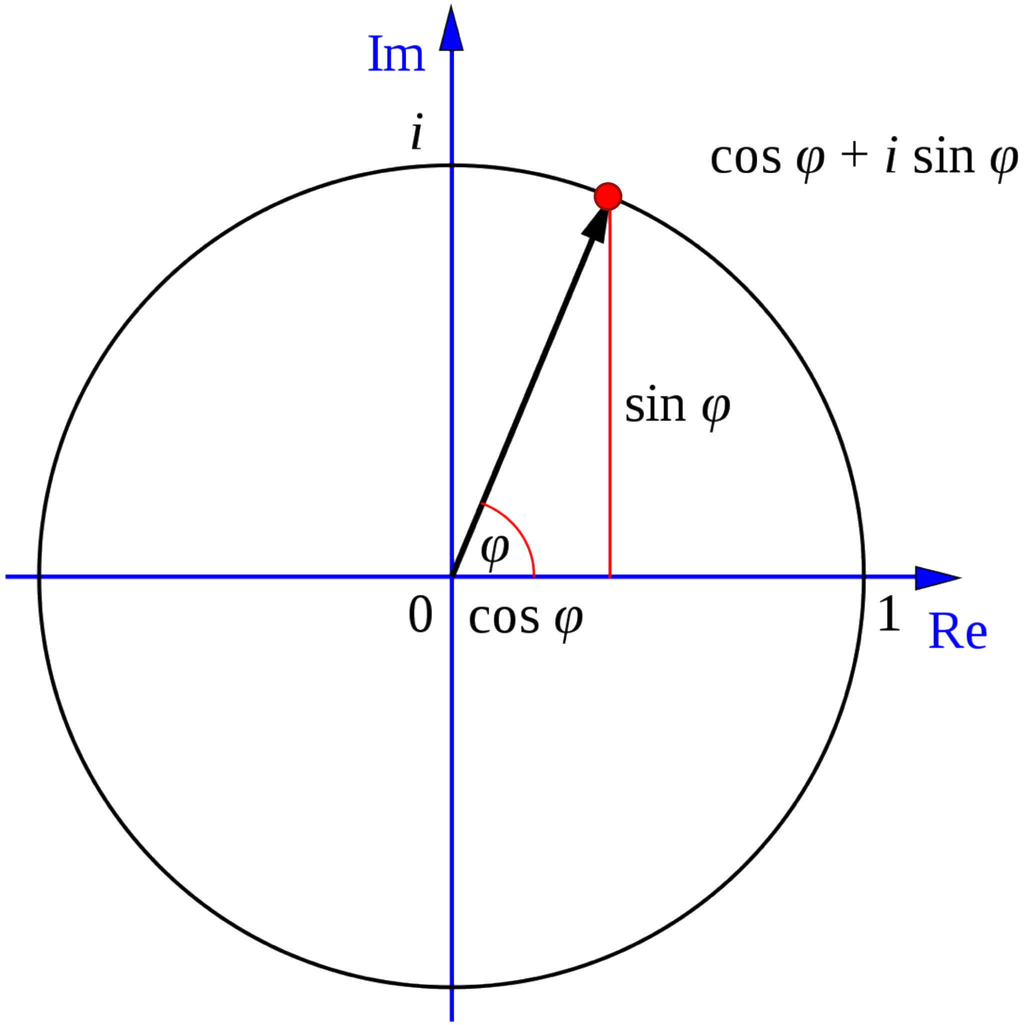
f *cycles per second*



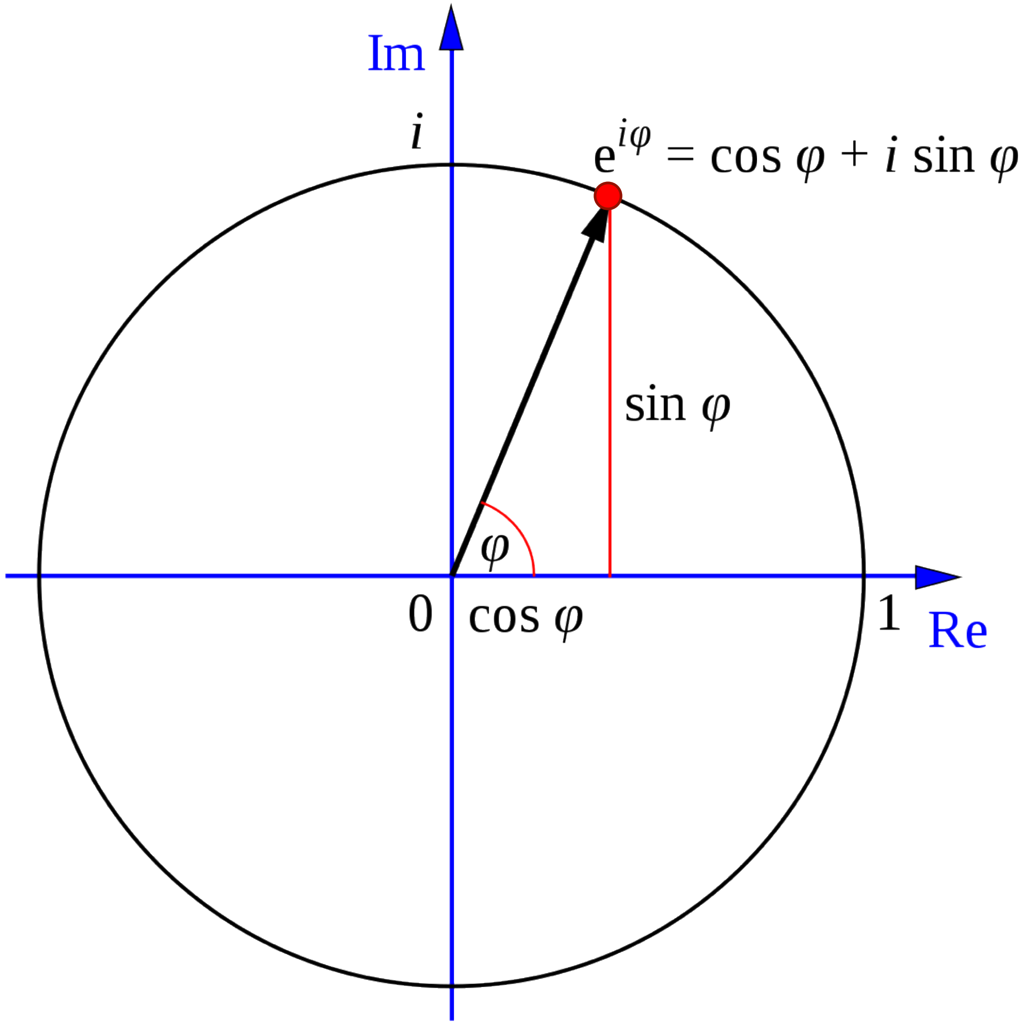
How to incorporate both Sine and Cosine in the equation?

$A\sin(2\pi f t + \phi)$ -- with initial/additional phase ϕ

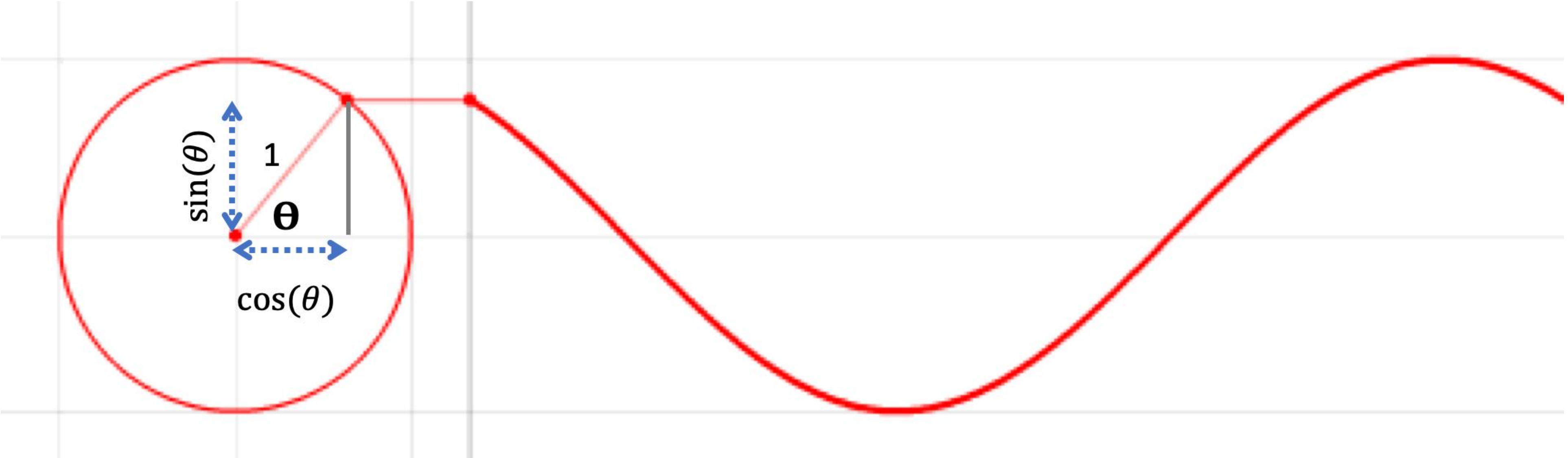
Complex Signals



Complex Signals



Complex Signals



$$cos(\theta) + j\ \sin(\theta) = e^{j\theta} = e^{j\,2\pi f t}$$

Complex Signals

- The most basic complex-valued signal is the complex exponential $e^{j\omega t}$.

$$\boxed{e^{j2\pi ft} = cos(2\pi\mathrm{ft}) + j\,\sin(2\pi\mathrm{ft})}$$

$$\boxed{e^{-j2\pi ft} = cos(2\pi\mathrm{ft}) - j\,\sin(2\pi\mathrm{ft})}$$

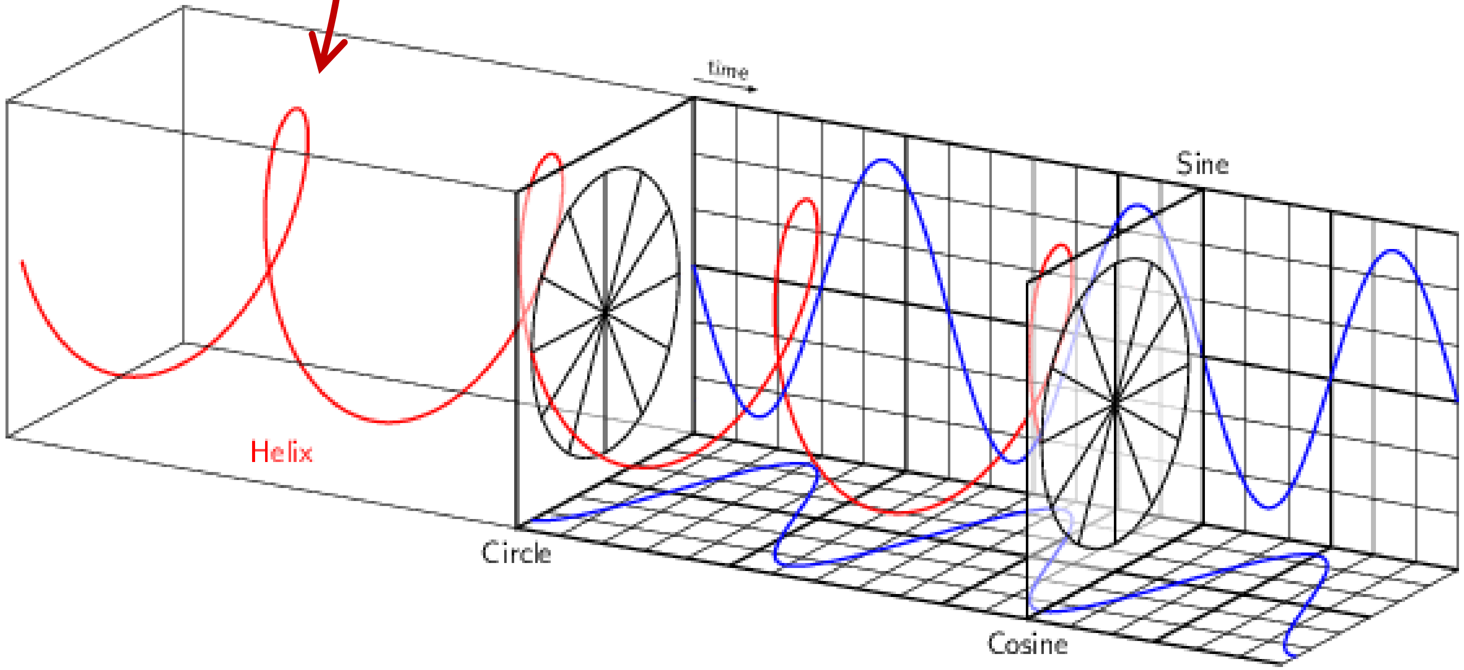
How about real sinusoids?

$$cos(\theta) = ?$$

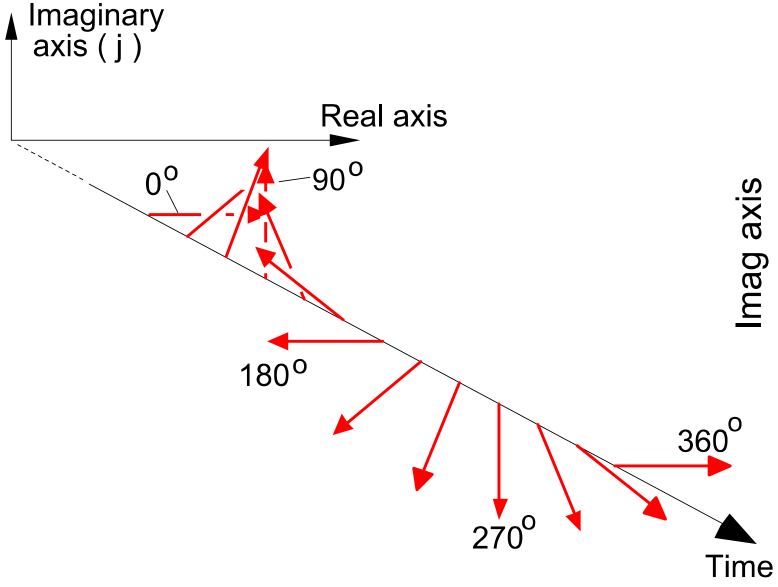
$$sin(\theta) = ?$$

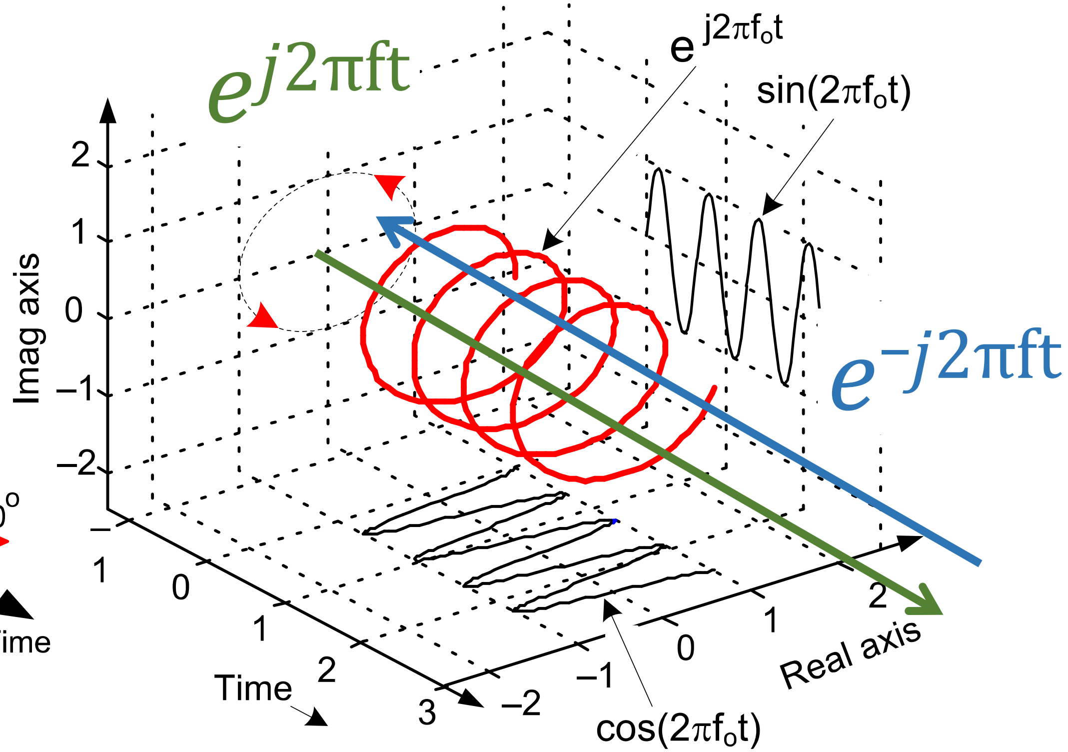
Complex Signals

$e^{j2\pi ft}$

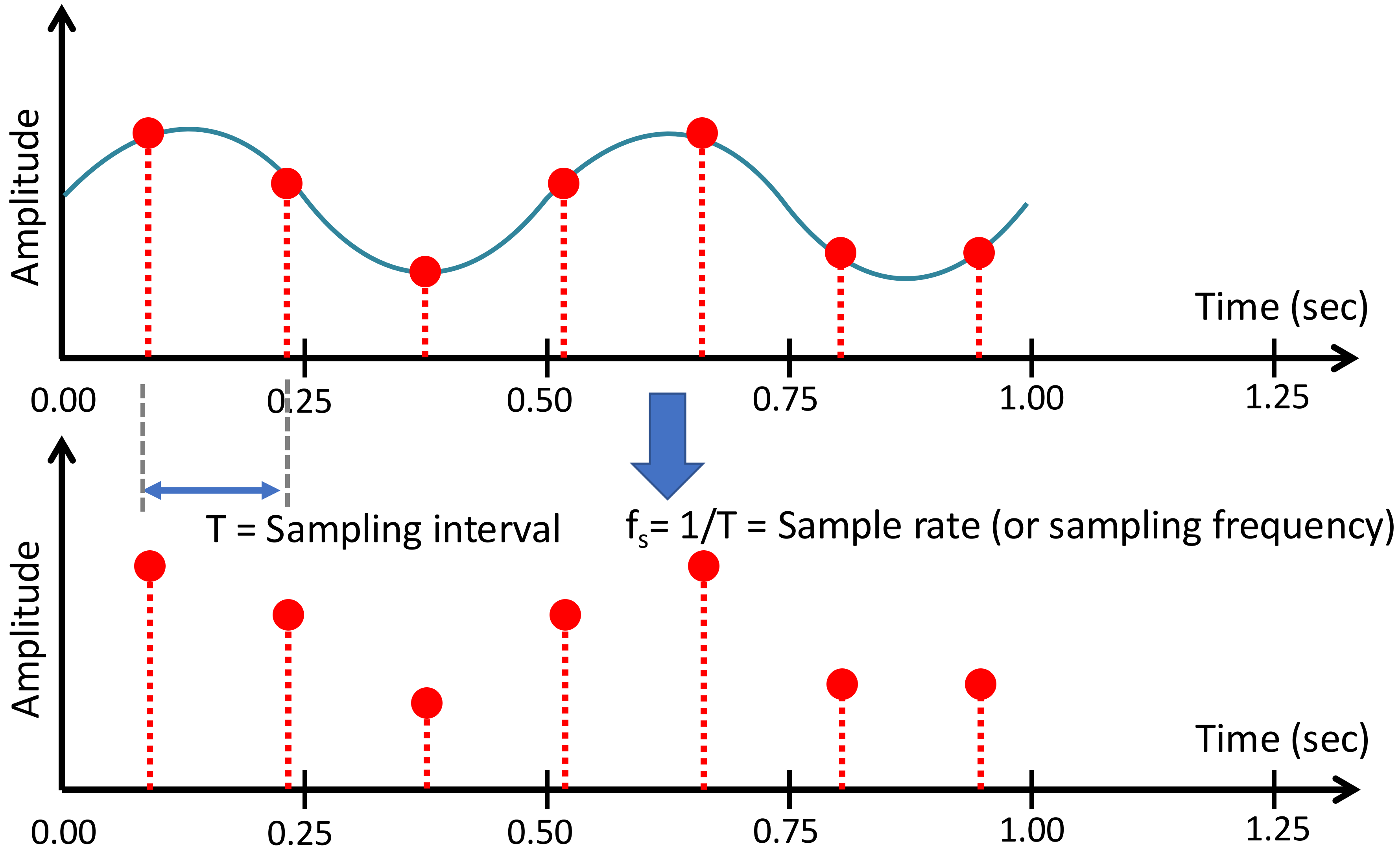


Complex Signals



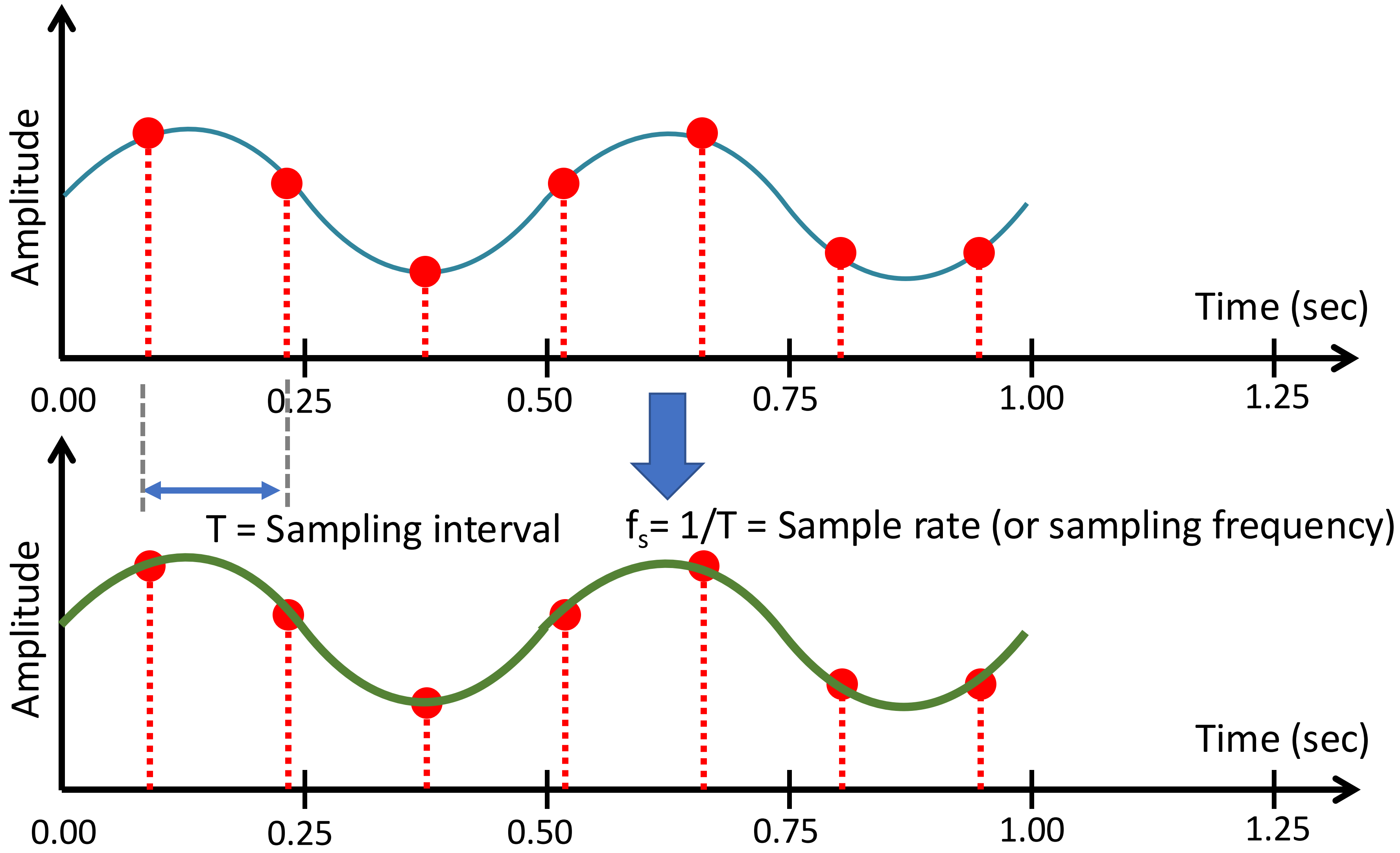


Sampling



Amplitude
Time (sec)
0.00
0.25
0.50
0.75
1.00
1.25
T = Sampling interval
f_s= 1/T = Sample rate (or sampling frequency)
Amplitude
Time (sec)
0.00
0.25
0.50
0.75
1.00
1.25

Sampling


Amplitude
Time (sec)
0.00
0.25
0.50
0.75
1.00
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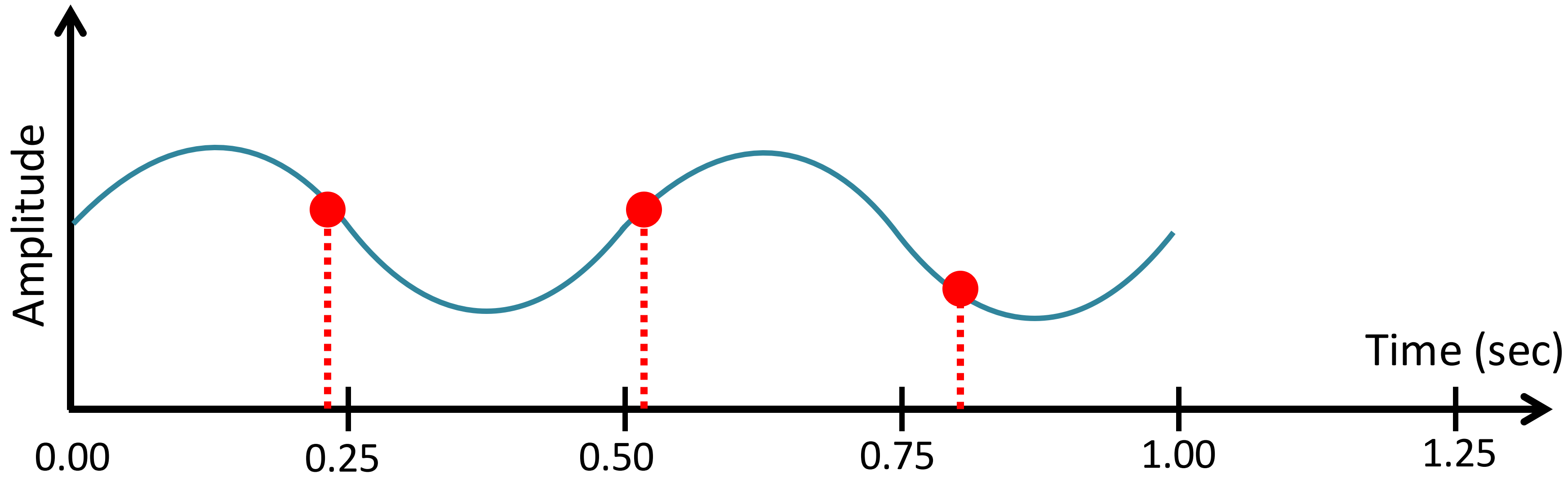
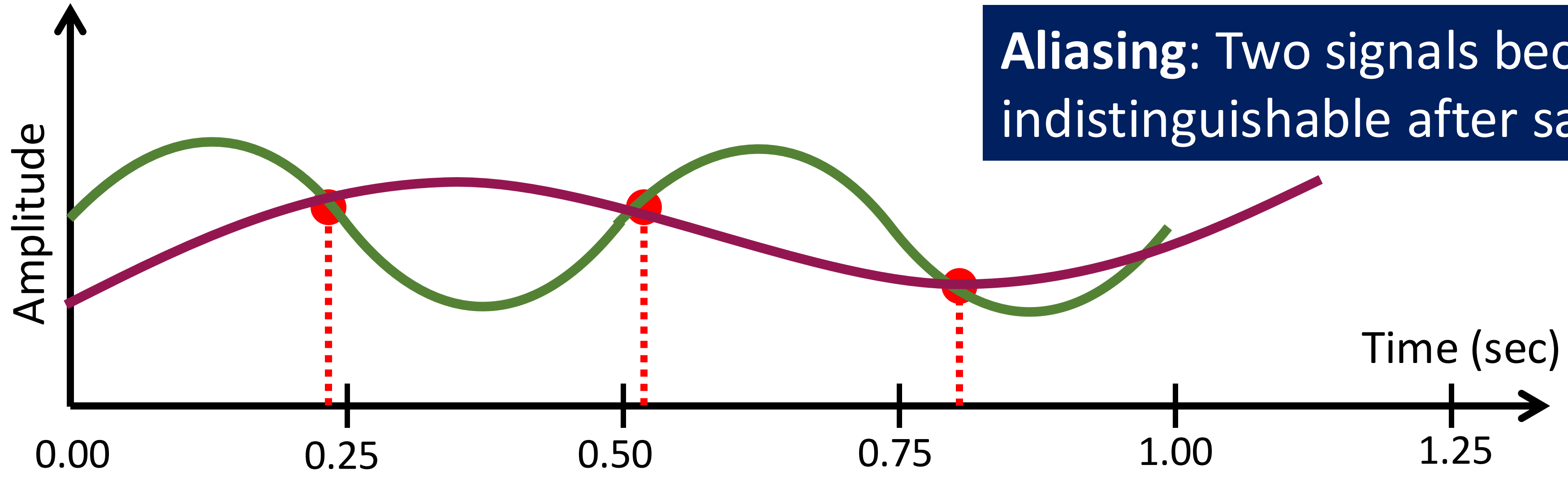
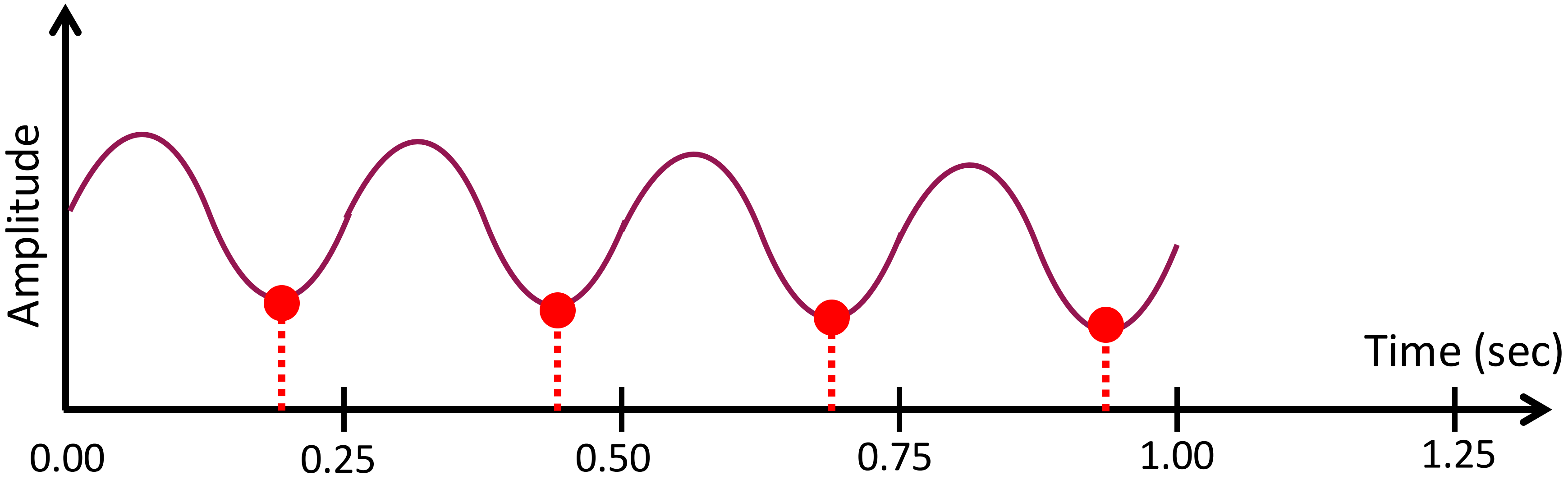
Sampling



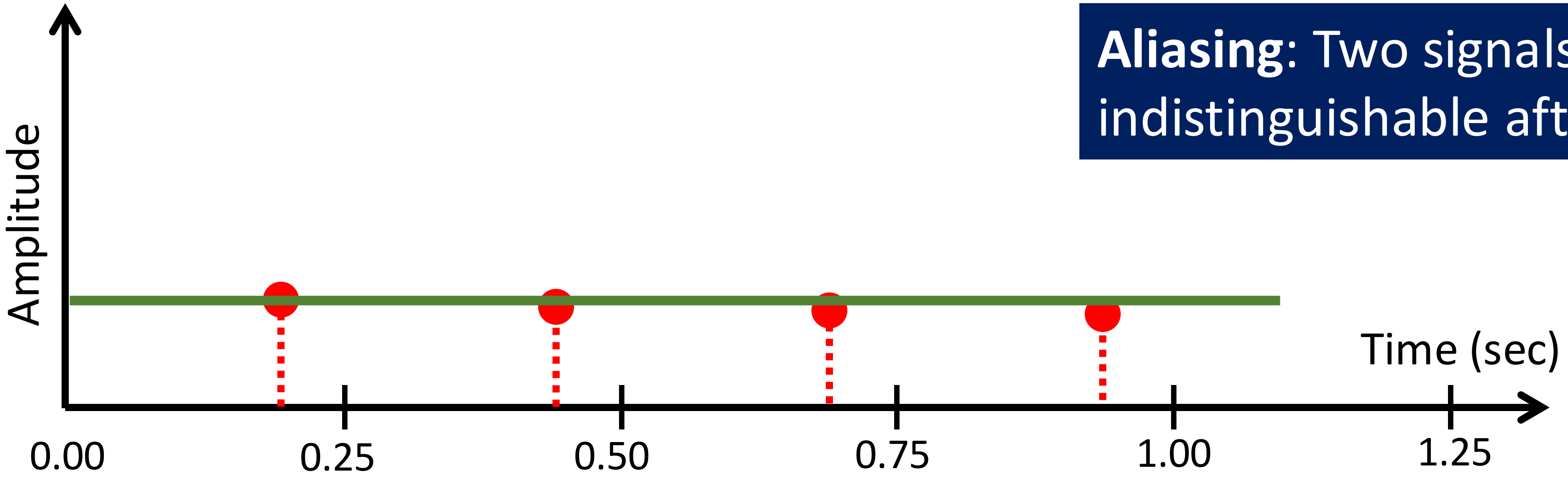


Aliasing: Two signals become indistinguishable after sampling

Sampling



Aliasing: Two signals become indistinguishable after sampling



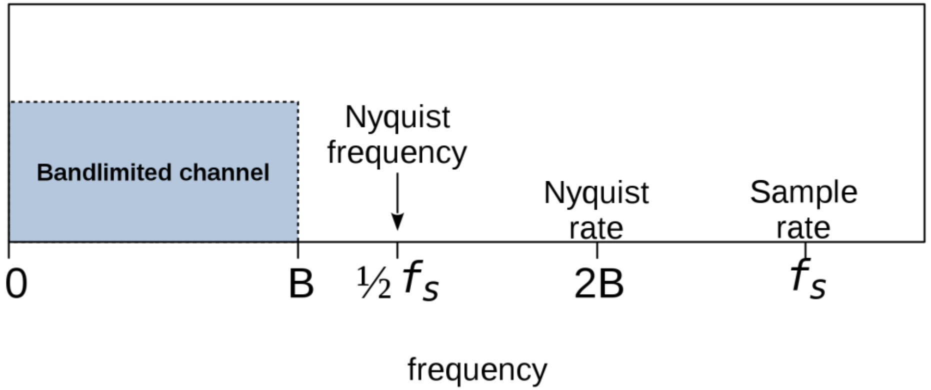
Sampling

- Nyquist sampling theorem:
    - In order to uniquely represent a signal x(t) by a set of samples, the sampling rate must be more than twice the highest frequency component present in x(t).
    - Sample twice per period!
- If sample rate is f_s and the maximum frequency of interest is f_{max}, then $f_s > 2f_{max}$

<u>Nyquist frequency</u>: Maximum alias-free frequency for a given f_s

<u>Nyquist rate</u>: Minimum sample rate for a given signal = twice the highest frequency

Relationship of Nyquist frequency & rate (example)



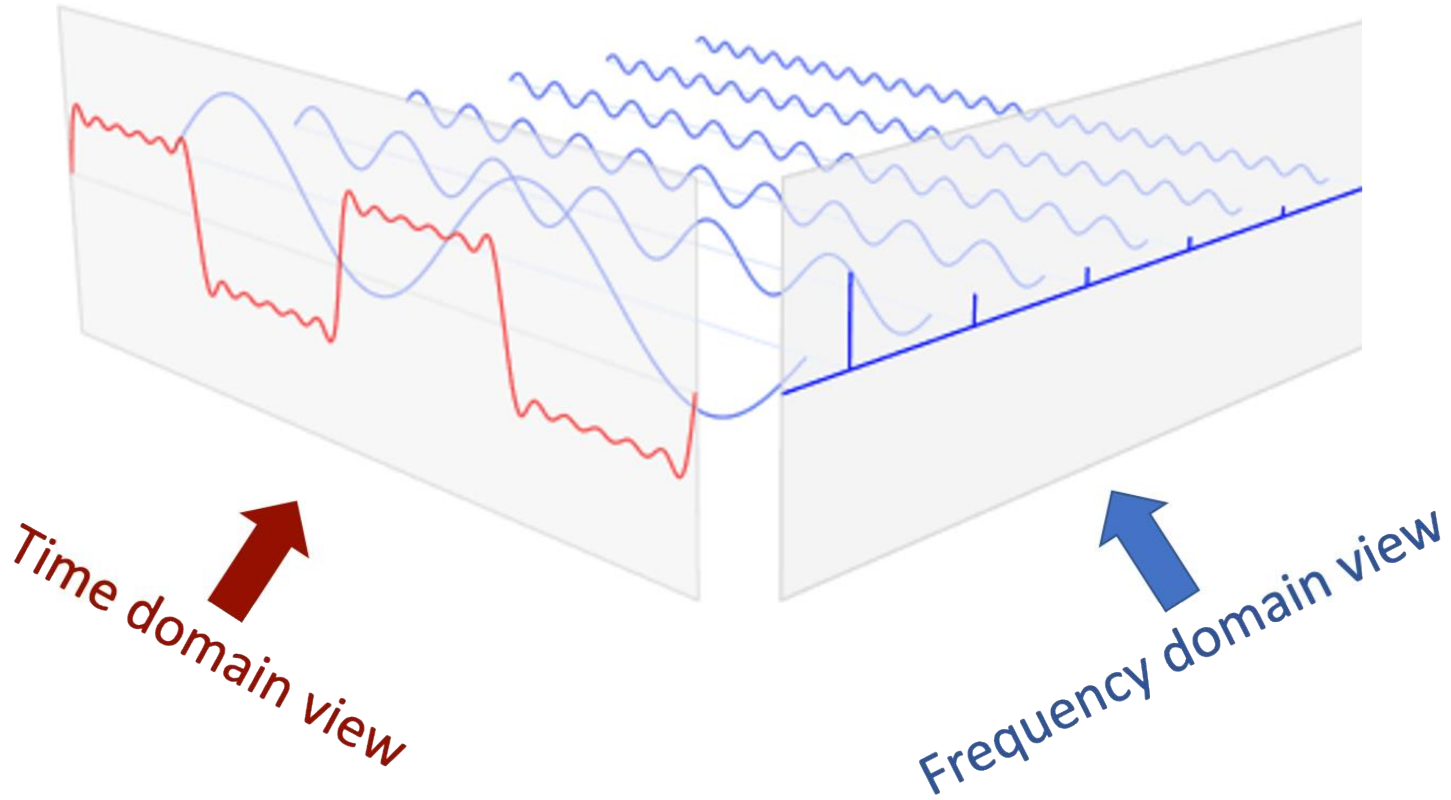
Aliasing in Real Life

https://www.youtube.com/watch?v=B8EMI3_0TO0

Aliasing in Real Life

https://www.youtube.com/watch?v=QOwzkND_ooU

Time and Frequency Domain



Why Frequency Domain?

Why Frequency Domain?


Smart Spoon, New Apps Help People with Parkinson's, Essential Tremors

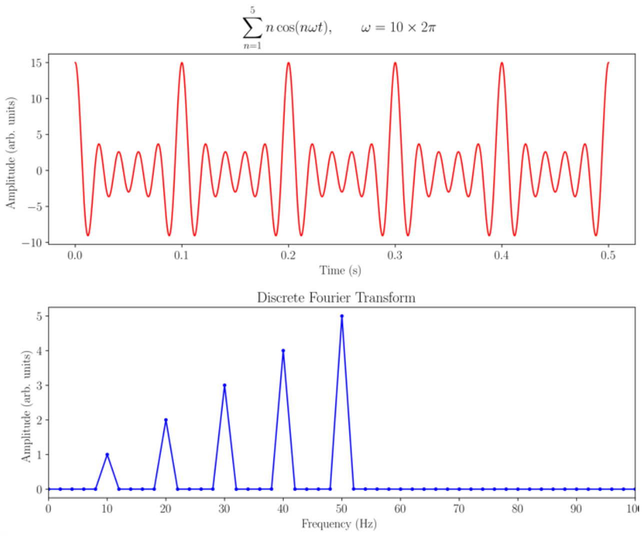
AUGUST 19TH, 2013 GAURAV NEUROLOGY, REHAB

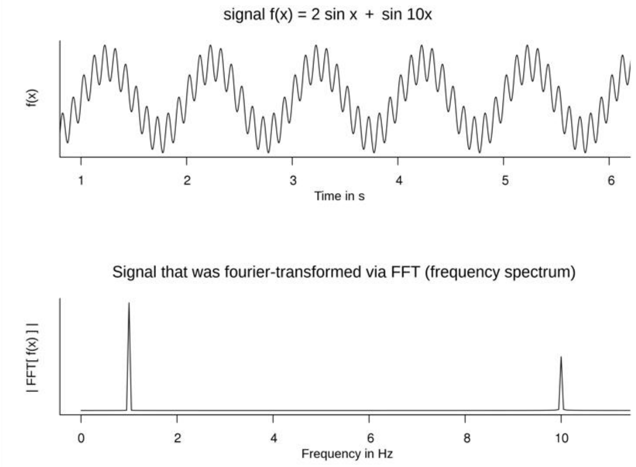
How to Get Frequency Domain?

- Discrete Fourier Transform (DFTs)
  - Convert time-domain signals $f(t)$ into the frequency-domain responses $F(\omega)$
  - Time-domain: $x[n], n = 0, 1, \ldots, N-1$
  - Frequency-domain: $X[k], k = 0,1, \ldots, N-1$
  - $\mathbf{X} = \mathcal{F}\{\mathbf{x}\}$

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{k}{N} n}}$$

- Fast Fourier transform (FFT)
  - Fast calculation of DFT: $O(N^2)$ to $O(NlogN)$



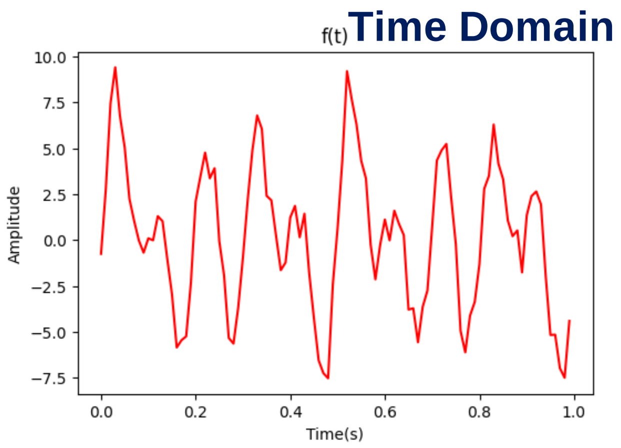


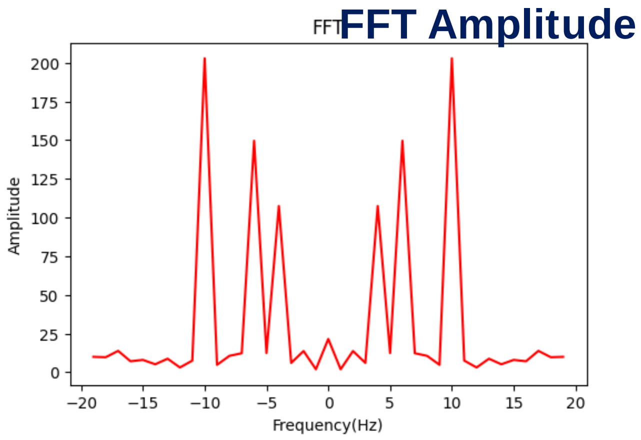
Understanding FFT (a bit)

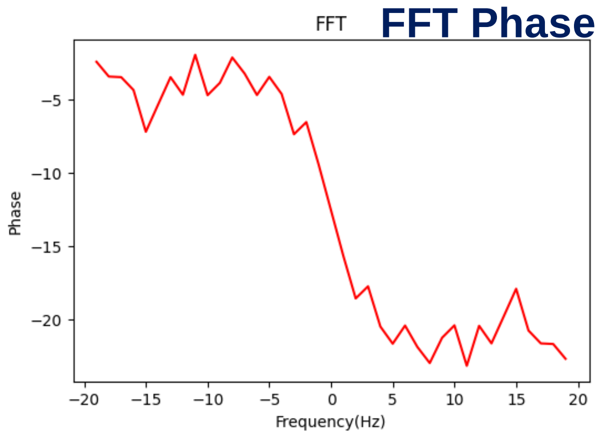
- What
  - Power spectrum of the signal
  - (Single-sided) Range: DC to $\frac{F_s}{2}$
- Why
  - Correlate x[n] with sin() and cos() signals at different frequencies

$$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi\frac{k}{N}n}=\sum_{n=0}^{N-1}x[n]\sin\left(\frac{2\pi kn}{N}\right)-j\sum_{n=0}^{N-1}x[n]\cos\left(\frac{2\pi kn}{N}\right)$$

- How
  - FFT Resolution of frequency bins: $\Delta f=\frac{F_s}{N}=\frac{1}{N\Delta t}=\frac{1}{T}$
  - The more observations/longer period, the better Δf
  - Frequency resolution vs. FFT resolution/bin width

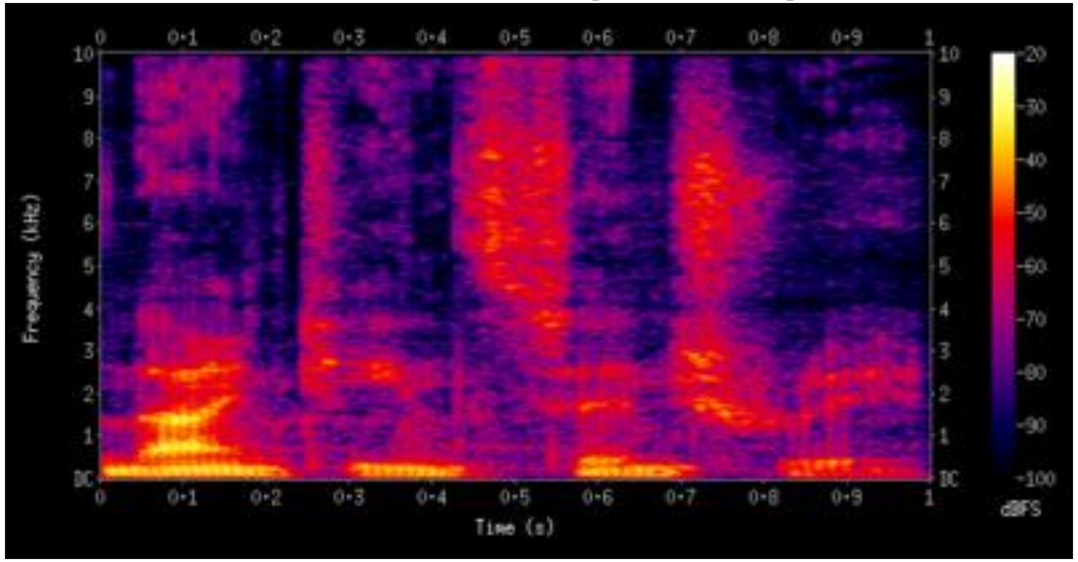






Time-Frequency Spectrogram

- For each time window a DFT is calculated and frequency intensity (amplitude) is represented with ”colors”.
- Repeat the calculation with a sliding window
- Short-Time Fourier Transform (STFT)



Questions?

- Thank you!